Retrofitting Fractional-Order Dynamics to an Existing Feedback Control System: From Classical Proportional-Integral (PI) Control to Fractional-Order Proportional-Derivative (FOPD) Control

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ABSTRACT

This paper presents a method of retrofitting an external controller into an existing feedback control system governed by a classical proportional-integral (PI) controller to improve the overall dynamics of the entire control system by benefitting from the advantages of fractional-order dynamics in the form of a fractional-order proportional-derivative (FOPD) controller. The retrofitting activity requires having access to the input and output signals of the existing control system ideally configured as a unity-feedback system. The difference between the input and output signals is then used as the input to an external controller having a fractional-order element. The design of the external controller is derived from the original values of the PI controller in the existing control system and the parameters of the desired FOPD controller to be used. A numerical example is presented in this paper to demonstrate how straightforward the process is. Discussion on some issues related to its hardware design and implementation is also presented in this paper.

Keywords: _control theory, control system analysis, fractional calculus, operational amplifiers, robust control_

1. INTRODUCTION

Fractional-order controllers (controllers with transfer functions of orders that are not necessarily of an integer) have proven to be effective in addressing various robust control issues and requirements in many applications over their integer—order counterparts [1]-[5]. They are able to simultaneously guarantee many robustness criteria given the appropriate architecture of the controller to be chosen [5]-[7]. Some of the robustness criteria are guaranteeing phase margin specifications, guaranteeing gain crossover frequency specifications, iso—damping (making the entire control system robust to gain variations in the plant), rejecting high-frequency noise, good output disturbance rejection, and steady—state error cancellation, to name a few. Simultaneous satisfaction of some, or all of these criteria, may be difficult, or yet, impossible for classical integer—order controllers such as the proportional-integral (PI) or proportional-integral-derivative (PID) controller and its family of controllers, which makes fractional-order controllers a better choice in many applications. This is evidenced by the increasing number of research publications on fractional—order controllers over the past decade.

There are infinitely many ways in creating a fractional-order controller, but one of the most common fractional-order controller used nowadays is the fractional-order proportional-derivative (FOPD) controller with the form

$$C_{FORD}(s) = K_P + K_D s^{\mu} \quad \textbf{(1)}$$

where, $K_P, K_D, \mu > 0$ result in an ordinary integer—order proportional-derivative (PD) controller found in many classical control texts.

The FOPD controller in Equation (1) can be interpreted as a 3-degree-of-freedom (3-DOF) controller because one can select the values of the three constants, i.e., $C = (K_P, K_D, \mu)$ to achieve certain results. The selection of the values of the constant triple C can be identified through any tuning rules published in literature for FOPD controllers in consideration of the type of plant to be controlled.

In an industrial control setting, the introduction of modern technology in a plant usually entails huge costs. Modernization of control equipment would require purchasing of new equipment and elimination of the old ones that are deemed to be obsolete as far as control benefit is concerned. In an attempt to minimize the total cost of modernization, one can pursue into retrofitting an external controller into the existing feedback control system instead, without making any significant modifications in the original, except for tapping signals at the input and output nodes.

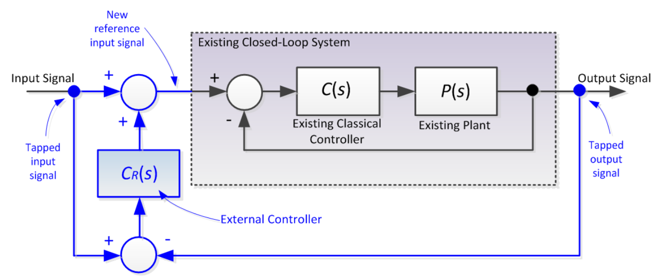

Figure 1. Architecture of an existing closed-loop system retrofitted with an external controller $C_R(s)$ by acquiring the difference between the input and output signals as the input to $C_R(s)$

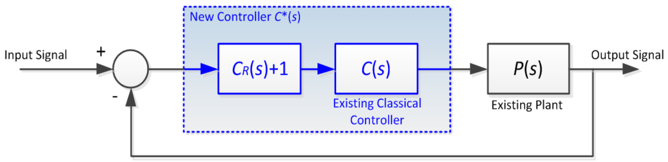

Figure 2. Equivalent block diagram of Figure 1 where the new controller $C_R(s)$ is defined as a function of the external controller $C_R(s)$ and the existing classical controller $C_R(s)$

Figure 1 presents an architecture in the form of a block diagram of an existing feedback control system in the unity-feedback configuration, where $C_R(s)$ is the existing classical controller and $P(s)$ is the existing plant. An external controller is then retrofitted by making changes in the input line of the existing system and tapping the input and output signals of the entire feedback control system to derive a certain error which is then inputted in the external controller $C_R(s)$ The new input reference signal to the existing control system is then the sum of the original input signal and the output of the external controller $C_R(s)$.

By applying block diagram algebra in Figure 1, an equivalent unity-feedback system can be derived as shown in Figure 2, where the new controller $C^*(s)$ becomes equivalent to

$$C^*(s) - (C_R(s) + 1)\, C(s) \qquad \textbf{(2)}$$

This means that if the external controller $C_R(s)$ is retrofitted into the existing control system, then the overall dynamics of the entire controller system can now be influenced by the equivalent controller $C^*(s)$ which is basically a function of the external controller $C_R(s)$ and the existing controller $C(s)$ In other words, by selecting the desired type of $C^*(s)$ in the form of FOPD, its influence in the existing control system can be theoretically guaranteed by determining the parameters of the retrofit external controller $C_R(s)$. This type of approach has been introduced initially by Sun [8] where the existing control system with either a classical PI or PID controller is modified to incorporate new integer-order dynamics to the incorporation of an external controller which is also integer-order by design. The work of Sun [8] was extended further in Gonzalez et al. [9] in 2013 to introduce fractional—order dynamics into the entire control system. In [9], the process of designing the external controller to be retrofitted was presented for three possible conversions: classical PI to fractional-order PI (FOPI), classical PI to fractional-order PID (FOPID), and classical PID to FOPID. The effectivity of the methodology was also demonstrated in a laboratory feedback control system composed mainly of analog circuitry in 2013 [10], in a magnetic levitation system in 2014 [11], and in a dc motor control system in 2016 [12].

This paper adds to the body of knowledge of fractional—order dynamics retrofitting.

Retrofitting Process and Architecture

In the retrofitting process, the main objective was to identify the parameters and specifications of the external controller $C_R(s)$. The following steps were to be taken in order to establish the parameters and specifications of $C_R(s)$:

Step 1: Given the model parameters of the plant, $P(s)$, using any applicable tuning method for FOPD controllers, e.g. [13]-[15], determine the constant triple $C^* = (K^*_P, K^*_I, \mu)$. It is important that $0 < \mu < 1$ is satisfied. (The asterisk "*" as superscripts means that the desired constants are to confuse with the constants K_P, and K_I from the existing classical PI controller.)

Step 2: Given the parameters of the existing classical PI controller and the FOPD constant triple, obtain the coefficients of the external controller transfer function of the form

$$C_R(s) = \frac{K_1 s^{\alpha} + (K_0 - K_P)s - K_I}{K_P s + K_I}, \quad (3)$$

where,

$$K_0 = K_P^* \quad (4)$$

$$K_0 = K_P^* \quad (5)$$

and

$$\alpha = 1 + \mu \quad (6)$$

Step 3: Design and implement the hardware system that would allow acquisition of input and output signals from the existing control system, and produce a new signal based on equation (3). The new signal from $C_R(s)$ will be added to the original input signal to serve as a new reference input signal to the existing control system.

The three—step process is rather straight forward and actually rely on the accuracy of the information gathered from the existing control system and the efficiency of the hardware retrofitted into the overall system. The basis of the external controller $C_R(s)$ in Equation (3) is explained as a theorem as follows:

Proposition: Let $C_R(s)$ in Equation (3) be the external controller where $K_1, K_0, K_P, K_I > 0$ and $1 < \alpha < 2$ following the architecture in Figures (1) and (2). The resulting equivalent controller $C^*(s)$ will be an FOPD of the form

$$C^*(s) = K_P^* + K_D^* s^{\mu} \quad (7)$$

where

$$K_P^* = K_0 \quad (8)$$

$$K_D^* = K_1 \quad (9)$$

and

$$\mu = \alpha - 1 \quad (10)$$

Proof: By incorporating equations (8)-(10) into equation (3), and then incorporating it again in equation (2), the equivalent controller obtained is as follows:

$$C^*(s) = (C_R(s) + 1)C(s)$$

$$= \left(\frac{K_1 s^{\alpha} + (K_0 - K_P)s - K_I}{K_P s + K_I} + 1\right)\left(K_P + \frac{K_I}{s}\right)$$

$$= \left(\frac{K_1 s^{\alpha} + (K_0 - K_P)s - K_I + K_P s + K_I}{K_P s + K_I}\right)$$

$$\times \left(\frac{K_P s + K_I}{s}\right)$$

$$= \left(\frac{K_1 s^{\alpha} + K_0 s}{s}\right) \times \frac{s^{-1}}{s^{-1}}$$

$$= K_1 s^{\alpha - 1} + K_0 \quad (11)$$

Incorporating equations (4) - (6) in equation (11) and re-arranging the terms results in

$$C^*(s) = K_1 s^{\alpha - 1} + K_0$$

$$C^*(s) = K_P^* + K_D^* s^{\mu}$$

A Numerical Example

Consider a hypothetical control system in the configuration of a unity-feedback where the plant is identified as the second-order system $P(s) = 1.52/s(0.4s + 1)$, which is taken from a dynamometer setup [13], while the controller is determined to be a classical integer-order PI controller with the transfer function $C(s) = K_P + K_I/s$, for $K_P, K_I > 0$. From the experiment in [13], assume further how to obtain the following robustness criteria: a guaranteed gain crossover frequency is at $\omega_c — 10$ rad/s, a guaranteed phase margin of $\varphi_m = 70^{\circ}$, and guaranteed robustness to gain variations in the plant, i.e. iso—damping. To obtain these desired robustness criteria, the desired new controller constant triple should have the following values: K^*_{μ} - 13.860, K^*_{μ} - 5.100, $\mu = 0.844$ as presented in [13].

Step 1 has already been fulfilled and it can be noted that the order of the new controller is between 0 and 1. The external controller can then be obtained from Step 2 which results in

$$C_R(s) = \frac{5.1s^{1.844} + (13.86 - K_P)s - K_I}{K_P s + K_I} \quad \textbf{(12)}$$

by investigating Equation (12) further, it can be seen that the coefficients of 2nd and 3rd terms of the numerator polynomial, and all the coefficients of the denominator polynomial are functions of the existing classical PI controller constants. The last step is to design the actual controller based on Equation (12).

Hardware Implementation of $C_R(s)$

There are many ways in implementing fractional—order controllers. One can choose to go for all-analog or utilize the flexibility of digital control. However, the discussion in this paper is restricted on analog hardware implementation using operational amplifiers (OP-AMPs).

It can be observed that the external controller ***$C_R(s)$*** in Equation (3) can be represented as a sum of three subsystems as

$$C_R(s) = \frac{K_1 s^{\alpha}}{K_P s + K_I} + \frac{(K_0 - K_P)s}{K_P s + K_I} + \frac{-K_I}{K_P s + K_I} \quad \textbf{(13)}$$

Each subsystem can be implemented using OP-AMP circuits where the feedback and arm elements in each OP-AMP circuit shall define the effective impedances that would mathematically represent each numerator and denominator expression in Equation (13).

The first expression of equation (13) has a fractional-degree numerator polynomial, which means that there is a fractional—order impedance in the OP—AMP circuit. Since resistor and capacitors are integer—order elements, there should be a special way to incorporate a device that can act as a fractional-order impedance. This device is called a fractional-order element (FOE) [7],[16]-[20] which can be approximated as an RC ladder. The final architecture for the first subsystem is defined as an RC ladder acting as the arm impedance and a parallel resistor—capacitor network at the feedback impedance.

The second and third expressions of Equation (13) are rather easier to implement. The second subsystem contains a parallel resistor—capacitor network as the feedback path element and a capacitor at the arm. Similarly, the third subsystem also contains a parallel resistor-capacitor network as the feedback path element, but with a resistor at the arm. The arithmetic Ladder circuit can also be implemented using OP—AMPs accordingly.

It is worth taking note though that the choice of OP-AMP is very crucial in its implementation especially its gain-bandwidth product (GDP) when operating at higher frequencies. If the operating frequency of the entire control system is at lower ranges, i.e. below 10 kHz, then the GDP of the OP-AMPs to be used may not be an issue.

Another note is that the input and output signals of the entire control system may be beyond the specifications of the OP—AMP circuit or may even be of a different physical quantity, e.g. air temperature or water pressure. Therefore, interfacing techniques may need to be applied accordingly.

CONCLUSIONS

This paper has presented a way to retrofit an external controller into an existing feedback control system to incorporate fractional-order dynamics. The retrofitting architecture would require acquiring signals from the input and output of the system where its difference is used as input to the external controller. The output signal of the external controller is then added to the original input signal of the system which then results in a new reference input signal.

The 3—step procedure is rather straight forward in terms of calculation but would require careful consideration when implementing using OP—AMP circuits. The numerical example shows that the procedure actually works as the architecture of the entire control itself from Figures 1 and 2 would automatically make the benefits of fractional—order dynamics inherent in a properly prepared case.

5. RECOMMENDATION

There is still more work to do in this area as the results in this paper and in previous publications just only give the tip of the iceberg and are yet far from the real—world industrial application. The authors recommend the conduct of more studies with re-searches dwelling more in the circuit implementation of the external controller either through analog, digital or hybrid means.

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REFERENCES

[1] C. A. Monje, B. M. Vinagre, V. Feliu, and YQ. Chen, “Tuning and auto-tuning of fractional order controllers for industry applications,” *Control Engineering Practice*, vol. 16, 2008, pp. 798-812.
[2] C. A. Monke, *Design Methods of Fractional order Controllers for Industrial Applications*, Ph.D. thesis, Escuela de IngenieríasIndustriales, Universidad de Extremadura, Badajob, Spain.
[3] B. Vinagre, I. Podlubny, L. Dorčák, and V. Feliu, “On fractional PID controllers: a frequency domain approach,” *IFAC Workshop on Digital Control. Past, Present and Future of PID Control*, Terrasa, Spain, pp. 53-58.
[4] I. Podlubny, “Fractional-order systems and $PI^{\lambda}D^{\mu}$ controllers,” *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 208-214.
[5] C. A. Monje, Y. Q. Chen, B. M. Vinagre, D. Xue, and V. Feliu, *Fractional Order Systems and Control: Fundamentals and Applications*, Advanced Industrial Control Series, London: Springer-Verlag, 2010.
[6] YQ. Chen, I. Petráš, and D. Xue, “Fractional order control-a tutorial,” *American Control Conference 2009*, Jun. 10, 2009, pp. 1397-1411.
[7] E. A. Gonzalez and I. Petráš, “Advances in fractional calculus: control and signal processing applications,” *16th International Carpathian Control Conference (ICCC)*, May 27, 2015, pp. 145-152.
[8] H. M. Son, “Retuning of PI/PID controllers based on closed-loop model,” *The AUN/SEED-Net Fieldwise Seminar on Control Engineering*, Montien Hotel, Bangkok, Thailand, Mar. 16-17, 2006, Session 1-4.
[9] E. A. Gonzalez, C. A. Monje, L. Dorčák, J. Terpák, and I. Petráš, “A method for incorporating fractional-order dynamics through PID control system retuning,” *International Journal of Pure and Applied Mathematics,* vol. 86, no. 4, 2013, pp. 593-605.
[10] L. Dorčák, E. Gonzalez, J. Terpák, I. Petráš, J. Valsa, and M. Žecová, “Application of PID retuning method for laboratory feedback control system incorporating FO dynamics,” *14th International Carpathian Control Conference (ICCC)*, May 26, 2013, pp. 38-43.
[11] A. Tepljakov, E. Petlenkov, J. Belikov, and E. A. Gonzalez, “Design of retuning fractional PID controllers for a closed-loop magnetic levitation control system,” *13th International Conference on Control Automation Robotics & Vision (ICARCV)*, Dec. 10-12, 2014, pp. 1345-1350.
[12] A. Tepljakov, E. A. Gonzalez, E. Petlenkov, J. Belikov, C. A. Monje, and I. Petráš, “Incorporation of fractional-order dynamics into an existing PI/PID dc motor control loop, *ISA Transactions*, vol. 60, Jan. 2016, pp. 262-273.
[13] HS. Li, Y. Luo, and YQ. Chen, “A fractional order proportional and derivative (FOPD) motion controller: tuning rule and experiments,” *IEEE Transactions on Control Systems Technology*, vol. 18, no. 2, Mar. 2010, pp. 516-520.
[14] HS. Li and YQ. Chen, “A fractional order proportional and derivative (FOPD) controller algorithm,” *2008 Chinese Control and Decision Conference*, 2008, pp. 4059-4063.
[15] Y. Luo and YQ. Chen, “Fractional-order [proportional derivative] controller for robust motion control: tuning procedure and validation,” *2009 American Control Conference*, 2009, pp. 1412 -1417.
[16] L. Dorčák, J. Valsa, E. Gonzalez, J. Terpák, I. Petráš, and L. Pivka, “Analogue realization of fractional-order dynamical systems,” *Entropy*, Special Issue on Dynamical Systems, vol. 15, no. 10, Oct. 7, 2013, pp. 4199-4214.

[17] A. C. Abad, E. A. Gonzalez, R. Y. Yap, and L. Dorčák, "Analog realization of a fractional-order element on 0.35μm CMOS technology," viXra:1401.0016, Jan. 8, 2014, http://vixra.org/abs/1401/0016.

[18] G. L. Abulencia and A. C. Abad, "Analog realization of a low-voltage sixteen selectable fractional-order differentiator in a 0.35μm CMOS technology," *Journal of Computational Innovations and Engineering Applications*, Jan. 31, 2017, pp. 33-43.

[19] G. L. Abulencia and A. C. Abad, "Analog realization of a low-voltage two-order selectable fractional-order differentiator in a 0.35μm CMOS technology," *2015 International Conference on Humanoid, Nanotechnolo gy, Information Technology, Communication and Control, Environment and Management (HNICEM)*, Dec. 9, 2015, pp. 1-6.

[20] E. A. Gonzalez, L. Dorčák, C. A. Monje, J. Valsa, F. S. Caluyo, and I. Petráš, "Conceptual design of a selectable fractional-order differentiator for industrial applications," *Fractional Calculus and Applied Analysis*, vol. 17, no. 3, 2014, pp. 697-716.